

S2 File. Analysis of the coinfection model using the full model

We investigate the properties of the co-infection model using the large model developed in the original HPV work's supplementary material from Murall et al 2015. The extended model includes the population of uninfected basal epithelial cells, X , which are born at rate $\lambda(t)$ and die at rate μ . HPV, W , interacts with uninfected basal epithelial cells, X , at rate ψ to produce infected cells, Y_1 . We assume that the infection is density dependent with ϕ representing the uninfected cell concentration where the infection is half-maximal. Below is the expanded co-infection model wherein we included the effect of tat , pV , with the birth of uninfected basal epithelium. The term $(1 + pV)\lambda(t)$ accounts for the production of epithelial cells that are susceptible for HPV infection. The rest of the co-infection dynamics are described in the main text.

The expanded co-infection model becomes:

$$\begin{aligned}
 \frac{dT}{dt} &= s - dT - (1 - \varepsilon_{RT})\beta TV, \\
 \frac{dI}{dt} &= (1 - \varepsilon_{RT})\beta TV - \delta I, \\
 \frac{dV}{dt} &= (1 - \varepsilon_{PI})N_1 \delta I - c_1 V, \\
 \frac{dX}{dt} &= (1 + pV)\lambda(t) - \mu X - \psi W \frac{X}{\phi + X}, \\
 \frac{dY_1}{dt} &= \psi W \frac{X}{\phi + X} - \varepsilon Y_1 - \mu Y_1 - a Y_1 E, \\
 \frac{dY_2}{dt} &= \varepsilon Y_1 + r \varepsilon Y_2 - \mu Y_2 - a Y_2 E, \\
 \frac{dW}{dt} &= \mu k(Y_1 + Y_2) - c_2 W, \\
 \frac{dE}{dt} &= \omega Y_2 E \left(1 - \frac{E}{K(T)}\right),
 \end{aligned} \tag{26}$$

with initial conditions with initial conditions $T(0) > 0$, $I(0) > 0$, $V(0) > 0$, $X(0) > X_1$, $Y_1(0) = Y_{10}$, $Y_2(0) = Y_{20}$, $W(0) = W_0$ and $E(0) = E_0$ where $t = 0$ is the time of co-infection.

Since, tat is known to play an important role in the disruption of epithelial tight junctions, thereby

facilitating the entry of HPV into the mucosal epithelium [30]. We compare the dynamics of model (26) against those of main model (5) for the same *tat* effect $(1 + p\bar{V}) = 2$.

When considering HIV induced immunosuppression in HIV/HPV co-infected individuals with different CD4+ T cells levels of i) $\bar{T} = 10^6$ cells per ml, corresponding to a healthy patient ii) $\bar{T} = 5 \times 10^5$ cells per ml, corresponding to average chronic HIV CD4+ T cell numbers [41]; iii) $\bar{T} = 3.3 \times 10^5$ cells per ml as in the HIV/HPV co-infection study [35]; and iv) $\bar{T} = 2 \times 10^5$ cells per ml, corresponding to AIDS; the extended model (26) has similar results as the reduced model (5) (see (A) in Fig. S2). Similarly, when we considered the effects of co-infection under the setup of the oral co-infection trial [35] for $pV = 1$ and $K(T) = 11.5$ cells (corresponding to CD4+ T cell concentration of $\bar{T} = 3.3 \times 10^5$ per ml) we found that both model (6) and (26) give similar results. In particular, HPV is cleared under cART conditions $\varepsilon_{RT} = 0.95$ and $\varepsilon_{PI} = 0.5$ and the timing of the clearance depends on two factors: the HPV stage and the level of CD4+ T cells at the start of cART (see (B) in Fig. S2).